antenna-transmission line analog

a key to designing and understanding antennas

Practical applications of this valuable tool in designing your own antenna systems

In the first part of this article it was shown in some detail how an antenna may be regarded as possessing both radiating and non-radiating properties.[1] I also discussed how the non-radiating TEM wave mode function may be used to convert the antenna into a special kind of rf transmission line. Using the antenna/transmission line concept in a way first made clear by Dr. Schelkunoff,[2] you can determine the mean or average characteristic impedance K of this antenna/transmission line and use it to calculate the antenna's input impedance behavior either at a single frequency, or over an entire band of frequencies.

In this section I will show you how to apply the antenna/transmission line analog concept to a number of typical antenna design problems which arise in both amateur and commercial radio communications. No higher mathematics is needed to use the analog key in the modified form presented here. **Table 1** references the basic equations used, based on everyday trigonometric functions. However, to remove any possible difficulty for the interested amateur who may be a bit rusty in ac math, not only are all examples fully worked out, step-by-step, but the Smith chart[3,4] is also used to clearly present the progression of events leading to each solution of the antenna design problem.

The best way to understand something new is to plunge in and start using it. Let's begin with an antenna which is of increasing interest to the amateur who is faced with shrinking backyard space or a nasty tempered landlord: the electrically small antenna. Many forms of this antenna have been around for a long time, yet it is a tricky little beast and requires surprising care in its design if you want to obtain a reasonable level of performance from it.

inductance loaded, electrically small antennas

When the electrical length, $2h_t^\circ$, of a doublet antenna is less than 180 degrees ($\lambda/2$), or the electrical length, h_t°, of a grounded monopole is less than 90 degrees ($\lambda/4$) at the operating frequency, the antenna is too

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short electrically to oscillate in a state of natural resonance. For a linear antenna to oscillate at its first (lowest) natural resonance, the series inductance and shunt capacitance *distributed per unit length* along its conductor (or conductors) must add up to the reactive sum $+jX_L + (-jX_C) = jo$ *ohms* in a way similar to that of lumped, series LC circuits.

It is easy to understand how the distributed series inductance comes about, even in a perfectly straight conductor, but the distributed shunt capacitance is less obvious. The shunt capacitance is distributed to ground from each tiny incremental length of conductor forming a grounded monopole, or from one arm of a balanced doublet to the opposite arm in the same bit-by-bit way. As shown in the first part of this article, the distributed shunt capacitance in the cylindrical linear antenna is *not* uniformly distributed along the conductor length: It is maximum nearest the antenna input terminals, and at a minimum at the end (or ends) of the antenna.

table 1. Mean characteristic impedance of cylindrical antennas.

Doublet $$K_A = 120\,[\log_e \frac{2(h_t')}{a} - 1]\ ohms \qquad 1$$

Grounded monopole $$K_m = 60\,[\log_e \frac{2(h_t')}{a} - 1]\ ohms \qquad 2$$

Input impedance, Z_{in}, of uniform characteristic impedance transmission lines (of length equal to h_n° where $n = 1, 2, 3 \ldots n$)

End open circuited

$(Z_s = \infty)$ $$Z_{in} = -jK_{A,M} \cot an\, h_n^\circ \qquad 3$$

End short circuited

$(Z_s = 0)$ Z $$Z_{in} = +jK_{A,M} \tan h_n^\circ \qquad 4$$

End terminated in complex impedance

$(Z_s = R \pm jX)$ $$Z_{in} = K_{A,M} \frac{(Z_s)\cos h_n^\circ \pm jK_{A,M} \sin h_n^\circ}{K_{A,M}\cos h_n^\circ + j(Z_s)\sin h_n^\circ} \qquad 5$$

Note: when $j(Z_s) = j(R \pm jX) = jR + jjX = \frac{jR + j^2X}{jR - j^2X}$, where: $+j^2 = -1$, $-j^2 = +1$

Useful Relationships

Inductance (henries)	$L = X_L/2\pi f$	*henries*	6
Capacitance (farads)	$C = 1/2\pi f X_c$	*farads*	7
Inductive reactance	$+jX_L = 2\pi fL$	*ohms*	8
Capacitive reactance	$-jX_c = 1/2\pi fc$	*ohms*	9

Where f is in Hz, L is in henries, and C is in farads

$Q\,(3\ dB)$ $$Q = f_o/(f_{high} - f_{low}) \qquad 10$$

When the linear antenna is too short electrically to oscillate naturally, two interesting things happen to its input impedance, $Z_{in} = R_{in} + jX_{in}$. First, that part of R_{in} representing the resistive-like radiation term R_r is smaller than that found in the naturally resonant antenna; that is, R_r is less than the 36-ohm radiation resistance of the quarter wave, grounded monopole or less than the 72-ohm radiation resistance of the half-wave doublet.

*The term "linear" is used here to distinguish between the electric antenna and its dual, the magnetic dipole or loop antenna.

At the same time, the input reactance, jX_{in}, *of the* electrically short antenna becomes capacitive. In the limit, when h_t° approaches zero degrees in length, the radiation resistance R_r approaches zero ohms and $-jX_{in}$ approaches infinity. Because the electrically short linear* antenna acts like a series-resonant circuit operating on the low frequency side of resonance, we can "force" it back into electrical resonance by doing something which cancels out the reactive part of its self impedance. One way to do this is to insert a lumped inductor (loading coil) in series with the antenna conductor. The reactance, $+jX_{coil}$, of the lumped loading reactor needed to force-resonate an electrically short antenna of given length, h_t°, will vary in magnitude, depending upon just *where* it is inserted in the antenna conductor. To investigate how the short antenna operates when the loading coil is placed *anywhere* along the antenna conductor, you must arrange the analogue transmission used to represent the antenna in such a way that it can handle any possible coil location.

Fig. 1A shows a *doublet* antenna of total length, $2h_t^\circ$, in the form of an analog coaxial transmission line. This is a model of the actual antenna, so think of the inner conductor of the analog line as the equivalent antenna conductor and the coaxial shield as the surrounding ground plane surface beneath the vertical monopole (or the influence of the capacitance to the other half of the doublet). The analog transmission line has a "uniform" characteristic impedance K_A, K_m, (the subscript A will be used for the doublet and m for the monopole). This analog transmission line $K_{A,m}$ is the mean or average value of the characteristic impedance of the cylindrical antenna which *actually varies* along its length. The mean characteristic impedance of the cylindrical antenna, given by Schelkunoff's **eq. 1** and **2** in **table 1**, forms the basis for the antenna/transmission line analog key method as it is used here.

The analog line "inner conductor" is broken in the exact center to form two balanced input terminals A-B (balanced to a fictional ground located midway between them); these are the doublet input terminals. You may view a doublet of length, $2h_t^\circ$, as being composed of *two identical monopole elements* each of length h_t°. As was shown in the first article, the input impedance of any doublet operating in free space is just twice the input impedance of *one* of its monopole elements operated against perfect ground. Because of this, from now on you can totally ignore the part of the doublet to the left of the dashed line G-G in **fig. 1A**, knowing that once you have completed the design of a grounded monopole which meets your performance requirements, you can convert it into a doublet by merely duplicating the monopole design and placing it on the other side of G-G.

The monopole has *unbalanced* input terminals A and G ground. Moving away from the terminal A toward the right, along the inner conductor of the analog line monopole, a break appears in the inner conductor at a distance h_1° from terminal A. On the other side of the gap the analog line continues on an additional length h_2° to its end. The end line terminals A_3, G_3 of the analog line are open circuited; exactly the same condition prevails at the tip or end of the actual monopole antenna which the line represents. The two analog line section lengths h_1° and h_2° always add up to the sum $h_1^\circ + h_2^\circ = h_t^\circ$. Finally, in **fig. 1A** you will notice that there are terminals A_1,A_2, A_3 at all locations where the analog line inner conductor is cut, and just opposite on the "shield" of the analog line are corresponding "ground" terminals G_1, G_2, G_3.

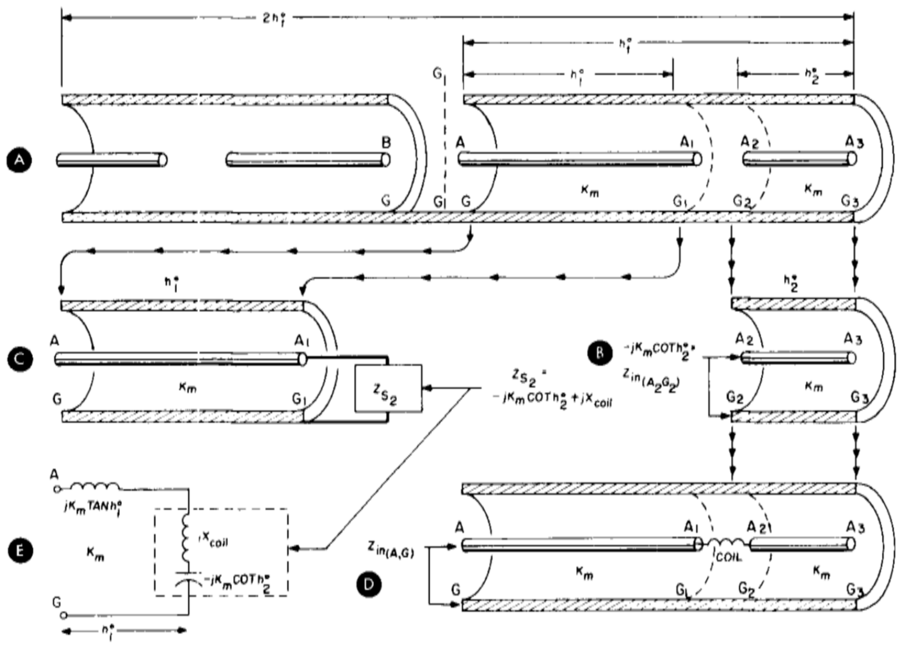


fig. 1. (A) Analog transmission line representation of a doublet antenna of length $2h_t^\circ$ composed of two identical monopoles of length h_t°. Each monopole is cut into additional length sections, h_1° and h_2°, for insertion of the loading coil. (B) shows input impedance of line end section h_2°. (C) shows line section h_1° terminated in load Z_{s2}. Short monopole with series loading coil is shown in (D). Equivalent circuit of inductively loaded monopole antenna is shown at (E).

Actually, in advanced work, any number of gaps or line sections can be used. In the equations of **table 1**, then, where you see a length denoted h_n°, you may substitute n equals 1, 2, 3 . . . n.

Why is the analog line broken in this way? It is to permit you to insert any "gadgets" such as loading coils, series capacitors, insulators, isolating parallel resonant LC "traps", and so forth which you may wish to use in your antenna designs. This is the "antenna-dissection-by-parts" technique mentioned at the end of the first part of this article. What has been said above applies to *any use* of the analog key in solving *any* antenna problem. In what follows, however, we will restrict ourselves to just two line sections, h_1° and h_2°, to explore the electrically short, coil loaded antenna.

design of coil loaded, electrically short antennas

Design objective: By applying the antenna/transmission line analog key to any electrically short cylindrical monopole antenna, you want to find the size of the loading coil needed to resonate the antenna at any frequency, *with the loading coil located at any distance* h_1° from the input terminals A, G.

Approach: You will have to occasionally refer to the equations of **table 1** in what follows. However, we want to do something other than just solve a string of equations and become bored. To do this we will use the Smith impedance chart shown in **fig. 2**. The Smith chart is sort of a motion picture version of the famous transmission line **eq. 5** in **table 1**. One important feature of the chart is that it lets you actually "see" what goes on as an impedance "moves" along the analog line (monopole), changing its magnitude as it travels to some particular place on the line.

Comparing **figs. 1A** and **2**, note that the "output terminals" A_3,G_3 at the very far end of the analog line (which represent the top or tip of the monopole) are located at the bottom of the Smith chart. These output terminals are located on the very edge of the inside rim scale at a point where $R + jX$ equals infinity. Such impedance magnitude corresponds to an open circuit; that is what the antenna tip sees as a "load" impedance. The two circular scales around the outside rim of the chart are marked off in wavelengths. In what follows, you will normally use only the one labeled "Wavelengths Toward Generator" (WTG). Just below this scale is one labeled "Wavelengths Toward Load" (WTL) which permits movement in the opposite direction along the analog line. To use these scales, all you have to do is to divide the distance in degrees you wish to move by 360 degrees to obtain distance in wavelength. The "generator" referred to is the transmitter, when it is thought of as being directly connected to the line input terminals A, G.

Because the modified analog key used here deals only with reactance (the resistive R part will be added later), all the impedances will be found on the very inside rim scale edge representing pure reactance. *Observe that all* inductive reactance $+jX$ is distributed around the right hand half of the chart circle; all capacitive reactance $-jX$ is around the left hand half. There is one more important matter: At the very center of the chart is a point which represents $Z = K_m\ (1.0 + j1.0)$ *ohms*. An impedance which reaches this particular spot on the Smith chart has a resistive R part equal in magnitude to that of the characteristic impedance K_m of the *particular* transmission line you are dealing with in the problem, and a zero reactance part jX. At point P, on the other hand, the impedance is $K_m(1.0 + j1.0)$, meaning its real part R and its *inductive* part jX are both equal in magnitude to the characteristic impedance K_m assigned to the chart.

To "enter" an actual impedance $R \pm jX$ into the Smith chart you must first divide the actual magnitudes of both R and jX by the K_m you have *assigned* to the chart for that particular problem. After you solve your problem on the chart and wish to "remove" your answer, you merely multiply both the real and reactive parts indicated on the chart by your assigned K_m value. Charts labeled in this way are said to be *normalized*. A normalized Smith chart can be used with transmission lines of *any* characteristic impedance you feel like assigning to the chart. This feature makes the normalized chart very handy to have around. Here, in working with the analog key, we will deal only with pure reactance $\pm jX$, all located on the very inside rim of the Smith chart, but in other applications an impedance point may appear anywhere on the chart.[5]

Up at the very top of the Smith chart in **fig. 2** is a point on the inside rim edge where $Z = K_m\ (0 + j0)$ *ohms*. This is "home plate" in the ball game we will play on this Smith chart "diamond." When the objective is to resonate an antenna at a desired frequency, we *always* have to reach "home plate."

design values

Let's start right off with a specific set of conductors for the coil loaded monopole antenna. Also, for convenience, we will use a frequency, $f = 1.97 \times 10^6$ *Hz* = *1.97 MHz* in the old 160-meter ham band. Because wavelength λ' equals *984/1.97 MHz = 499.49 feet* at our selected frequency, even "short" antennas are physically large on 160 meters.

Conductor length: The monopole conductor length can be anything you wish as long as it is less than ninety degrees, so let's choose a total conductor length, h_t° = *33 degrees*. With *1.97 MHz* as the selected frequency, $33^\circ/360^\circ$ = *0.0917 wavelength*. At this point in the design we will deliberately *not make any distinction **whatsoever between the physical and electrical length of the monopole conductor***. The reason for this "error" will be explained later on. Taking this viewpoint, the 33 degree monopole conductor length becomes: h_t' = *499.49 feet x 0.0917 = 45.80 feet*.

Conductor diameter: Conductor diameter may be any size you want. However, since that conductor diameter just might have some effect on the loading coil size needed to resonate the short 33 degree monopole, select three different conductor radii: a_1 = *0.05 inch*; a_2 = *0.5 inch*, and a_3 = *1.5 inch*. To keep units the same in subsequent calculations, change these three conductor radii to feet, getting: a_1 = *0.004*, a_2 = *0.04*, and a_3 = *0.125 feet*.

Loading coil location, h_1°: Since we are exploring loading-coil placement, let's choose: base loading, center loading, and "almost" top loading. Now, by the convention given in **fig. 1**, base loading a monopole means that h_1° = *zero degrees*, because no break or gap would exist along the inner conductor for this case. With h_1° = *zero*, that makes $h_2^\circ = h_t^\circ$ = *33 degrees*. For the center loading case h_1° becomes $\frac{1}{2}h_t^\circ$, or in our case, *16.5 degrees*. This choice also makes the remaining length h_2° equal *16.5 degrees*, too, because the sum, $h_1^\circ + h_2^\circ$, *always* has to equal h_t° = *33 degrees*. Finally, we will explore the almost top loading case, h_1° = *32 degrees*, leaving h_2° equal to *1 degree*.

Monopole mean characteristic impedance, K_m. Because we are going to use three different conductor radii with the 45.80-foot conductor length, we will obtain three different values of the mean characteristic impedance, K_m, to represent each of the monopoles, even though h_t° = *33 degrees* is fixed in this case. From **eq. 2** of **table 1**, for the cylindrical monopole, first work out $K_{m(1)}$ representing the first conductor radius, a_1 = *0.004 feet*:

$$K_{m(1)} = 60[(\log_e \frac{2(h_t')}{a}) - 1] = 60[(\log_e \frac{245.8}{0.004}) - 1]$$

$$= 60[(\log_e 22900) - 1] = 60(10.04 - 1)$$

$$= 542.32 \text{ ohms}$$

Substituting the remaining two conductor radii, a_2 and a_3 into **eq. 2**, with h_t° = *45.80 feet*,

$$K_{m(2)} = 404.18 \text{ ohms}$$

$$K_{m(3)} = 335.80 \text{ ohms}$$

Case 1. Base-loaded monopole. Coil in series with base terminal A at monopole input (h_1° = *zero degrees*). Enter the normalized Smith chart (**fig. 2**) at its "output" end terminals A_3G_3 located at *0.250* λ on the WTG scale (the monopole top). Since the loading coil is located all the way down at the base of the 33-degree long monopole, we must travel that entire distance along the analog line to reach this "coil location" point. When we get there the chart will give us the normalized value of the capacitive input reactance, $-jX_{in(A,G)}$ of our monopole for this case. Since the "starting point" in the journey along the line is designated *0.250* λ on the chart, we must *add* the distance $33^\circ/360^\circ = 0.092\lambda$ to that of our starting point to get the total distance:

$$0.250\lambda + 0.092\lambda = 0.342\lambda$$

Therefore, 0.342λ is the "stopping point" on the WTG scale. If you very carefully draw a straight line from the center of the chart to intersect this 0.342λ point on the WTG scale, it cuts through the *- 1.54* magnitude on the capacitive reactance scale. The rest is easy. The reactance *- 1.54* means simply $-jK_m(1.54)$ *ohms*. This is the *normalized* reactance $-jX_{in(A,G)}$ of any *cylindrical monopole* whose *electrical* length is 33 degrees. To change this normalized capacitive reactance to represent the actual reactance $-jX_{in(A,G)}$ of the three particular monopoles under discussion, you need only multiply that normalized value by each of your three calculated K_m values. But wait! For *only* the case of a loading coil located at the monopole input terminals, the size of the loading coil reactance is the *same* as the absolute magnitude of $-jX_{in}$, if you just change the sign of the reactance to a plus. You can immediately determine the reactances of all three loading coils. They are simply $+jK_m\ (1.54)\ ohms$:

$$+jX_{coil(1)} = +j\,835.10\ ohms \qquad (K_m = 542.32\ ohms)$$
$$+jX_{coil(2)} = +j\,622.22\ ohms \qquad (K_m = 404.18\ ohms)$$
$$+jX_{coil(3)} = +j\,517.13\ ohms \qquad (K_m = 335.80\ ohms)$$

It's a good thing we chose three different conductor radii; the calculations indicate that conductor diameter certainly does change the size of loading coils in an electrically short antenna of the *same* electrical length $h_t°$. Not only that, but "fat" conductors need smaller loading coils than thin conductors to produce resonance in short monopoles.

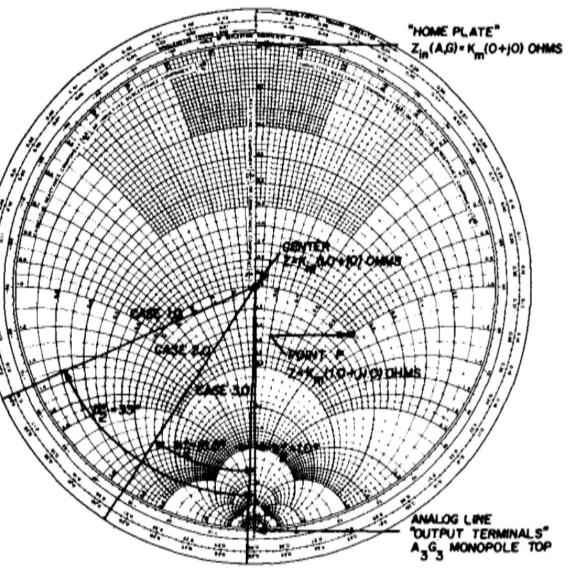


fig. 2. Normalized Smith chart which gives solution for loading coil reactance necessary to resonate a grounded monopole ($h_t° = 33°$) of any K_m and three locations, $h_t°$, for the loading coil: Case 1, base loading; Case 2, center loading; Case 3, loading 1° from top of monopole.

Case 2. Center-loaded monopole: $h_1° = 16.5°$ $h_2° = 16.5°$ Because we start at the top of the antenna/analog line, the distance to the coil location is always $h_2°$. In the center loaded case, then the distance of travel will be $16.5°/360° = 0.046\lambda$. Adding that to the 0.25λ starting point, the stopping point on the WTG line from the center of the chart through this new point, 0.296λ, you'll find a larger normalized value of capacitive reactance, -3.38, which you now know to be $-jK_m(3.38)\ ohms$. Before you get ahead of me and begin happily changing the sign of that reactance multiplied by the three values of K_m, and generating a new table of loading coils, *hold it*! The $h_1°$ is no longer zero degrees! This capacitive reactance $-jK_m(3.38)\ ohms$ is really $-jX_{in(A_2G_2)}$, as shown in **fig. 1B.**

Looking over to **fig. 1E**, you'll see that this "capacitor" produced by monopole analog line section of length $h_2°$ is connected in series with the loading coil reactance $+jK_{coil(5)}$ and this *combination* becomes load impedance $Z_{s(2)}$. Now $Z_{s(2)}$ is seen in **fig. 1C** to be connected across the output terminals, A_1-G_1 of the lower monopole analog line section of length $h_1°$. In **fig. 1E**, the lower line section $h_1°$ will "insert" an additional inductive reactance $+jK_m(tan\ h_1°)$ in series with the monopole analog line input terminals A-G. If you cancel out *all* the capacitive reactance at the loading coil gap which is produced by the upper line section $h_2°$, by inserting a loading coil reactance $+jX_{coil}$ of equal magnitude, you will go "skidding past" the *home plate* at $Z_{in(A,G)} = K_m(0 + j0)\ ohms$, and end up with a $Z_{in(A,G)}$ reactive part jX_{in} just equal in magnitude to this $jK_m tan\ h_1°$ produced by the lower monopole analog line section. What you really need is the "resonant" reactance condition

$$jX_{in(A,G)} = jK_m(X_{coil}) + (-jK_m X_{in}(A_2G_2)) + jK_m\ tan\ h_1° = jK_m(0.0), \qquad \textbf{(1)}$$

where $-jK_m X_{in(A_2G_2)}$ is really just $-jK_m cot\ h_2°$ from **eq. 3** in **table 1. Eq. 3** is the thing we are solving with the Smith chart when we enter it at the antenna top and go traveling along the WTG scale a distance $h_2°$ to the "stopping" point. When you want high accuracy, use **eq. 3**, because you can't read the Smith chart that closely. It is like a slide rule in this respect, but good enough for preliminary design. Looking at **eq. 1** above, you can now see how to determine the size of the loading coil; just leave $+jX_{coil}$ on the left side, and put everything else on the other side of the equals sign. Then,

$$jX_{coil} = jK_m(cot\ h_2°) - jK_m(tan\ h_1°)\ ohms \qquad \textbf{(1-1)}$$

Since you just obtained $jK_m(cot\ h_2°) = jK_m(3.38)$ from the Smith chart, and you know $h_1°$ also equals $16.5\ degrees$, and $16.5° - 0.30$

$$jX_{coil} = jK_m(3.38) - jK_m(0.30) = +jK_m(3.08)\ ohms$$

Now you can go ahead and generate a list of center loading-coil reactances:

$$jX_{coil(1)} = +j\,1670.35\ ohms \qquad (K_m = 542.32\ ohms)$$
$$jX_{coil(2)} = +j\,1244.87\ ohms \qquad (K_m = 404.18\ ohms)$$
$$jX_{coil(3)} = +j\,1034.26\ ohms \qquad (K_m = 335.80\ ohms)$$

Notice that the loading coils for the center-loaded case in the $h_t° = 33\ degree$ monopole are $3.08/1.54$ or two times as large as the loading coils needed to base load the *same antenna* to the *same* frequency. Also again, "fat" conductors still require smaller loading coil size. Does this coil growth trend continue with increasing $h_1°$? If so, what is the "rate" of increase? To find out, go on to the "almost top loaded" case where $h_1°$ is 32°, and $h_2°$ is only 1 degree.

Case 3. Almost top-loaded monopole $h_1° = 32°$ and $h_1° = 1.0°$. Having to go only $1.0°/360°$ equals 0.002λ, add that to 0.250λ and obtain 0.2520λ as the stopping point on the WTG scale. However, now you cannot read the normalized capacitive reactance $-jK_m(X_{in})A_2,G_2$ – the Smith chart reactance scale is too cramped in this region, which is approaching infinite values. No problem! Simply use **eq. 3** from **table 1** and to find $-jK_m(X_{in})A_2G_2$:

$$-jK_m(X_{in})A_2G_2 = -jK_m(\cot 1.0°)$$
$$= -jK_m(57.29)\ ohms$$

Flipping the sign of the answer above to plus (and knowing that tangent $h_1° = tangent\ 32° = 0.625$) we plug these values into **eq. 1-1**:

$$+jX_{coil} = jK_m(57.29) - jK_m(0.625)$$
$$= +jK_m(56.66)\ ohms$$

Our final table of coil reactances follows

$jX_{coil(1)} = +j\ 30730\ ohms$ $(K_m = 542.32\ ohms)$
$jX_{coil(2)} = +j\ 22902\ ohms$ $(K_m = 404.18\ ohms)$
$jX_{coil(3)} = +j\ 19028\ ohms$ $(K_m = 335.80\ ohms)$

Clearly, above the center loading-coil location, coil reactance grows rapidly, at an exponential rate. At $1.97\ MHz$, **eq. 6** from **table 1** gives the inductance of these coils for a location only one degree from the antenna top as

$$Coil\ (1) = 2.48 \times 10^{-3}\ henry = 2.48\ mH$$
$$Coil\ (2) = 1.85 \times 10^{-3}\ henry = 1.85\ mH$$
$$Coil\ (3) = 1.53 \times 10^{-3}\ henry = 1.53\ mH$$

Coils of this size are large enough to serve as decoupling chokes in the power supply leads in your transmitter! Incidently, before turning to other matters, note that if you want to translate the short, coil-loaded monopole design technique to any *other* ham band; select an $h_t°$ other than 33 degrees; or use monopole conductor radii other than 0.050, 0.50, or 1.5 inch, you will have to calculate K_m again for the new case. Once this is done, you can quickly proceed as shown here to find the reactance of the needed loading coils at any height, $h_1°$. Also, as you will later see, in terms of antenna efficiency it is best to reduce the size of the loading coil; the largest diameter conductor you can use will help in this respect.

Now that you have seen how the size of the loading coil changes with its placement in the monopole, you may well ask, "Why should I use a coil location other than $h_1°$ equals zero degrees? Why not just base load short monopoles?" To answer these critical questions we must turn to the problem of antenna radiation resistance in electrically short antennas.

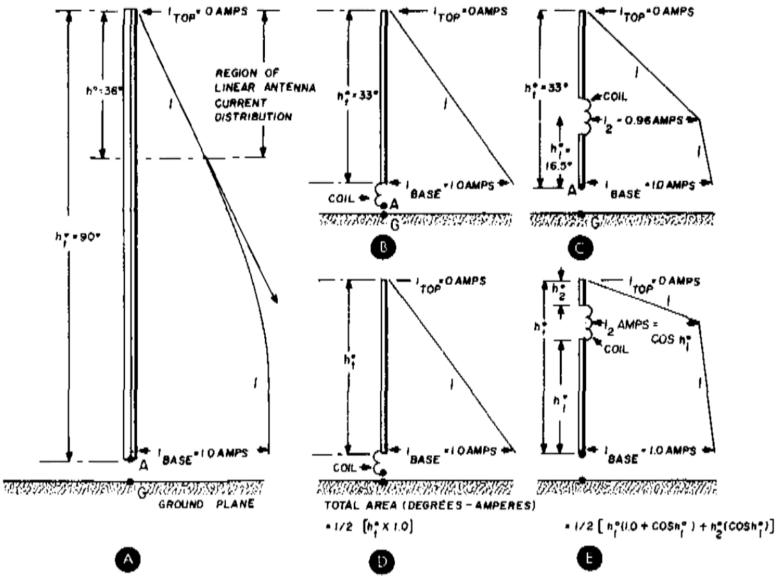


fig. 3. (A) Sinusoidal current distribution along naturally resonant quarter-wavelength monopole antenna. (B) Linear current distribution for base-coil loaded monopole, $h_t° = 33\ degrees$. (C) Same height monopole with center-coil loading. (D) Total current area for base coil loaded monopole, $h_t°$ less than 36°. (E) Total current area for short monopole antenna with loading coil at distance $h_1°$ from base. All monopoles are assumed to be resonant.

radiation resistance of electrically short antennas

Up to now we have been merely plugging in $R = 0$ ohms in our antenna input impedance, $Z_{in(A,G)} = R_{in} + jX_{in}\ ohms$. The modified antenna/transmission line analog key we've been using here gives us only the reactive part. Don't blame Dr. Schelkunoff for that. His elegant mode theory model *gives both* R and jX answers for $Z_{in(A,G)}$, but it employs some high power math. However, it turns out that the answers we are getting for jX_{in} compare quite accurately to those obtained by his more refined equations. That's fine. But how do we get the resistive part, R?

To add the R part to our answer, we assume R equals the antenna radiation resistance R_r. Then we just look it up in graphs giving R_r as a function of antenna length $h_t°$. Such curves are found in the literature.[6] This is fortunate, because calculating radiation resistance, R_r, for any antenna length $h_t°$ and current distribution is not exactly *child's play*. So it seems we have it all wrapped up: Just look up R_r for our particular antenna length $h_t°$ in degrees, and plug it into $Z_{in} = R + jX$. Unfortunately, it's not quite that easy. For antennas whose length is greater than $h_t° = 36$ *degrees*, the published curves give only the radiation resistance of electrically short antennas *which do not use reactive loading*. Therefore, you must be able to calculate the R_r of the reactively loaded, electrically short antennas when $h_t°$ is less than 36 degrees. When you do that you will see why base loading is not the optimum way to resonate electrically short antennas.

Fig. 3A shows the idealized, sinusoidal current distribution along a *naturally* resonant, quarter-wavelength vertical monopole antenna operating against a ground plane. If you plot the function, sine $h_t°$, on graph paper, starting with zero degrees at the top of the monopole (where current in this case is zero), you will find that within the first 36 degrees you do not obtain a curve at all; for all practical purposes, you will get a straight or linear line plot. Only at lengths greater than 36 degrees does the graph plot begin to look anything like a curve. Because of this fact you can deal with the current distribution of monopole antennas, less than 36 degrees high as *straight-sided* geometric figures. Accordingly, **fig. 3B** shows a base-loaded monopole of height $h_t° = 33$ *degrees*, with a triangular current distribution. In **fig. 3C** the same height antenna, center loaded, still has a straight-sided current distribution in the shape of a trapezoid.

Now, if you measure antenna section heights $h_1°$ and $h_2°$ in electrical degrees, and the current amplitude in amperes, the total area of the current distribution can be expressed in units of degree-amperes. One of the cruder (but good enough ways) to explain radiation resistance is to say that Nature only "sees" the antenna's total "exposed" current area. Based on this engineering "viewpoint" you can write an expression* which gives the radiation resistance, R_r, of electrically short ($h_t°$ less than $36°$) monopole antennas operating over a ground plane as

$$R_r = 0.01215\, A^2 \qquad \textbf{(1-2)}$$

where A is the total exposed antenna current area in degree-amperes.

For a doublet of length $2h_t°$ in free space, composed of two short, reactively loaded monopoles of length $h_t°$, just double the R_r magnitude calculated from **eq. 1-2**. **Fig. 3D** provides a formula for finding the triangular shaped, degree-ampere area of base-loaded monopoles over ground; **fig. 3E** gives the formula for the case where the loading coil is located a distance $h_1°$ from the input terminals A-G. It is important to note that, in all antennas shown in **fig. 3**, the antenna base current is *always* one ampere. This is the relative current amplitude which you must use because the radiation resistance of an antenna does not change with the different amplitudes of actual currents when you vary transmitter input power. For reasons which space does not permit me to go into here, when the antenna is *resonated* by a loading coil placed at a distance $h_1°$ from the base input terminals, the *relative* current I_2 at the coil position has an amplitude of $I_2 = cosine\ h_1°$.

**Eq. 1-2 is based on the theory of monopole moment.*

Using the above information, you can now see what happens to radiation resistance in short antennas as $h_1°$ is changed while antenna height, $h_t°$, is fixed. For example, consider your antenna, $h_t° = 33$ *degrees* when it is base loaded. From **fig. 3D**, the current area is then

$$A = \tfrac{1}{2}\,(33° \times 1.0\ ampere)$$
$$= 16.5\ degree\text{-}amperes$$

From **eq. 1-2**:

$$R_r = 0.01215\,(16.5\ degree\text{-}amperes)^2 = 3.3\ ohms$$

For the case where the loading coil was moved to $h_1° = 16.5°$, to center load it, $I_2 cosine\ 16.5° = 0.96$ *amperes relative*, giving us the area from **fig. 3E** as,

$$A = \tfrac{1}{2}[16.5°\,(1.0 + 0.96) + 16.5°\,(0.96)]$$
$$= \tfrac{1}{2}[32.34 + 15.84]$$
$$= \tfrac{1}{2}[48.18] = 24.10\ degree\text{-}amperes$$
$$R_r = 7.06\ ohms$$

You can see that center loading *increased* the radiation resistance by a ratio of *7.06/3.3 or 2.14 times*. The ohmic loss produced by a radial wire ground plane, insulator leakage, soil current resistance, and the like, might have a realistic ohmic resistance magnitude of 10 ohms. Antenna radiation efficiency, N, is related to the ratio between antenna radiation resistance, R_r and total environmental ohmic loss, R_Ω, as

$$N = \left(\frac{R_r}{R_r + R_\Omega}\right) 100\ (per\ cent) \qquad \textbf{(1-3)}$$

With ohmic loss equal to 10 ohms, the base-coil loaded monopole ($h_t° = 33°$) would yield an efficiency of,

$$N = \left(\frac{3.3}{3.3 + 10.0}\right)100 = 24.8\ per\ cent$$

The same height monopole, when center loaded, would yield

$$N = \left(\frac{7.06}{7.06 + 10.0}\right)100 = 41.4\ per\ cent$$

Not only is more input power radiated from the center-loaded monopole, but since $Z_{in} = R_r + R_\Omega + jX$, the magnitude of input impedance at resonance is increased, which makes impedance matching with a base network less of a headache. Right about here someone is going to remember the case where the loading coil was only 1 degree down from the top. Using the relations given, $R_r = 10.93$ *ohms*. This looks promising – increased R_r means increased efficiency, right? I am sorry I must dash cold water over this happy discovery. If you could raise the current-degree ampere area of the 33-degree monopole to the indicated total area *without introducing additional ohmic loss*, you could increase efficiency because R_r would increase toward its theoretical limit. The theoretical limit in reactively-loaded short monopole (or dipole) antennas is *four* times that of the base-loaded R_r (or doublet feedpoint loading), and is based on a perfectly rectangular current distribution. However, you can't approach this limit except in very short antennas, and *never* in the case of any *coil-only* loaded short antenna.

The words "without introducing any additional loss", however, lets the air out of this little balloon. Remember the rapid increase in loading-coil reactance (and thus inductance) as it climbed to higher and higher $h_1°$? You might think a loading coil is just hacked out of any commercial length of coil stock you have laying around the shack. Not true. An antenna loading coil must be tailored to your radiating system if you want to obtain maximum efficiency.

loading coils in electrically short antennas

Space limitation prevents me from going too deeply into the critical matter of loading-coil design. Still, being terse, I'll try to cram in some vital facts. By now your intuition must tell you that loading coil loss should be kept to the barest possible minimum. That clearly means use of a high value of coil Q. Unfortunately, coil ohmic loss R_L is related to coil Q by the expression

$$R_L = \frac{X_{coil}}{Q} \; ohms \qquad (1\text{-}4)$$

Eq. 1-4 says that, as coil reactance grows with increasing $h_1°$, coil Q must be increased to retain this *minimum R_L coil loss. Here, also, is a sobering thought:* the coil shape factor for maximum Q in a single layer coil is represented by a coil *diameter twice that of coil length*.[7] This ratio is not especially critical, but you should stay pretty close to it. So you see that as reactance increases, demanding more coil turns in increasing length, the physical diameter of the loading coil keeps increasing as it climbs the short antenna. Another important matter is the insulation holding the turns to the correct pitch. The material used for coil insulation should have the lowest possible dielectric loss factor, and a *minimum* amount of it should be employed. Moisture from the weather should be kept from the exposed loading coil by a sealed dielectric housing, because the slightest moisture film-bridging between turns, across the insulation, will reduce coil Q. Because of their low value of radiation resistance, the input current to short antennas is considerably larger than it is in, say, a naturally resonant quarter-wavelength monopole at the same input power. The antenna input (base) current, in terms of its actual magnitude, is

$$I_{in} = \sqrt{\frac{Pi}{R_r + R_o}} \; amperes \qquad (1\text{-}5)$$

where Pi is power input in watts.

Consequently, provisions must be made to handle the calculated current amplitude in the coil conductor. Voltage stress (voltage drop) across the loading coil is given by

$$V_{coil} = (I_c \times X_{coil}) \; volts \qquad (1\text{-}6)$$

Where I_c is the current flowing through the coil at its location $h_1°$ in the antenna.* If I_c is in rms amperes, multiply V_{coil} by *1.414* to get peak volts of stress.

Practical antenna engineers, adding this all up, feel that the center-loaded case is about optimum in terms of a trade off between increase in radiation resistance to offset environmental ohmic loss, and design of the loading coil within practical and economic limits.

A final word about tuning a short, coil-loaded antenna: Once you have achieved the high Q coil you need to resonate the little monster to the frequency in the band you select, don't try to change its inductance by means of tapped turns, turn sliders, or shorted-out turns to tune the antenna around the band. The resulting change in coil shape factor, and the eddy currents which *will circulate in the unused turns, will murder coil Q.* Instead, in fixed site home station versions retune the impedance matching base network you have to use anyway to lower transmission line vswr. In electrically short, mobile antennas use a telescoping section of the antenna conductor *above* the loading coil to establish resonance of the antenna at frequencies above or below f_o. Finally, I hope that you will now look at the long, skinny loading coils wound on massive insulating forms which you may see used in some short antenna designs with a healthy bit of skepticism. However, with use of the analog key, you can now design any coil-loaded, short antenna which comes to mind.

calculated coil size accuracy

Real antennas of any kind are *not* lumped circuits. Their extensive fields reach far out into the so called near-zone region surrounding them to "feel and sense" their electromagnetic environment and to react to it. They do this by automatically adjusting their electrical characteristics to always satisfy Nature's laws of *minimum energy balance*. Electrically short antennas are the most sensitive of all, due to the much larger storage of energy within their near-zone region. To an antenna, each station site, each mobile vehicle, differs considerably from another.

It is due to this peculiar nature of the antenna, in contrast to that of lumped circuits, which makes it impossible for man to devise a calculation method which can *exactly* predict in the design stage all the changes which will occur in a real antenna operating in some actual electromagnetic environment. Therefore, working antenna engineers make the best calculations they can at the office desk, then put on their hard hats and go out to the antenna measurement laboratory and prune their creations exactly to size. This problem is least at microwave frequencies, and at its worst at frequencies in the low- and high-frequency bands. In the case of the electrically short antenna, this means pruning calculated coil size. But you can use an "insurance policy" here because first, you don't want to start out with a coil which is too small, and secondly, you don't want to do too much

*Multiply actual I_{in} by cosine $h_1°$ to find I_c when the coil is located at a point higher than the antenna's base-input terminals.

pruning. How does the analog key satisfy these two design requirements for the practical designer?

Recall that when you started to select antenna conductor length for the electrically short antenna you did not make a correction between conductor physical length and its electrical length. I said we would regard them at the start as equal to one another. It turns out that unless an antenna conductor is of zero diameter, its electrical length is always greater than its physical length. If you examine **eq. 3** in **table 4**, you will realize that if such is the case, all the magnitudes of capacitive reactance calculated from it, or its solutions via the Smith chart, are somewhat smaller than those obtained. That means that all of the calculated coil reactance values are too large.

In matter-of-fact, their size error on the plus side of tolerance is in inverse proportion to the mean antenna characteristic impedance K_m you calculated them to match. Thus, the coil calculated for $K_m = 335.8$ *ohms* would resonate the 45.8 foot tall monopole to a frequency below *1.97 MHz* in the 160 meter band; the coil for $K_m = 542.3$ ohms would resonate the same height monopole closest to, but still a bit below *1.97 MHz*. Not too much below *1.97 MHz* (or whatever frequency you designed for) but enough that you may safely prune the coil *up in frequency* to resonate in your own particular environment. I do that with a grid-dip oscillator which is *loosely* coupled to the grounded coil/monopole combination.

At the end of our next example, you will see how to make the correction between conductor's physical and electrical length when you need to. This, however, will be in working with naturally resonant antennas which are least sensitive to environment.

frequency bandwidth of antennas

Design objective. You want to design a naturally resonant antenna in such a way that the vswr on the transmission line feeding the antenna does not exceed a specified maximum value at any frequency within the limits $f_{high} - f_{low}$ of a given amateur band.

Such antenna design will provide you with an antenna into which a modern transmitter, using a pi-network tank circuit, will easily load full power at any spot in the band. Most amateur transmitters are designed to load full power only into a transmission line vswr of 2.8:1 or less.

Foreward to problem. Remember that I said that the input reactance, jX_{in}, of an antenna of fixed length changed far more rapidly in magnitude with a change in transmitter frequency than the resistive part, R_{in}. The resistive part of antenna input impedance, in fact, changes so little over the frequency width $f_{high} - f_{low}$ of any presently assigned high-frequency amateur band that we may make an engineering *approximation* and view R_{in} as a constant over the frequency width $f_{high} - f_{low}$.

With R_{in} assumed constant over the frequency width of any amateur band, let's take up the case of a naturally resonant quarter-wavelength monopole antenna, remembering that we can always combine two such monopole designs into a single half-wave doublet if we wish.

The radiation resistance, R_r, of a naturally resonant quarter-wavelength monopole over perfect ground is equal to 36 ohms. If we select the natural resonant frequency, f_o, of the monopole to be that of the band center, then its input impedance at, f_o, will be $Z_{in(A,G)} = 36 + j0$ *ohms*. When we tune the transmitter vfo above and below f_o, the linear monopole antenna will exhibit the impedance characteristics of a series-resonant circuit. That is, it will exhibit a $-jX_{in}$ capacitive reactance in series with R_r at frequencies below that of f_o, and a jX_{in} inductive reactance at frequencies above f_o.

When the absolute magnitude of the reactive part of the antenna input impedance becomes just equal to the magnitude of the resistive part, antenna designers refer

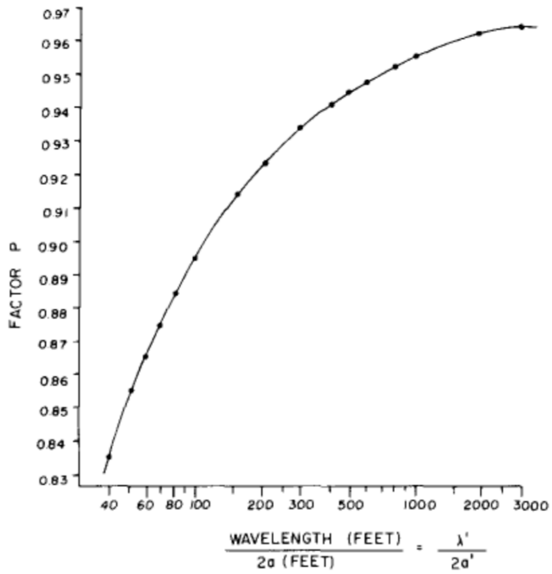


fig. 4. Correction to obtain height, h_t (feet) equal to 90 electrical degrees at resonant frequency. To use this chart use the following procedure:

1. **Calculate wavelength from** $\lambda = 984/f\,(MHz)$.
2. **Divide monopole conductor diameter,** $2a$ **(feet) into** λ **(feet).**
3. **Enter graph to obtain length correction factor,** P.
4. **Use** $246P/f_o$ **(MHz) to obtain resonant monopole length,** h_t **(feet).**

For example, what is the correct height at 3.6 MHz for a monopole constructed of 3-inch (0.25 foot) aluminum pipe?

$\lambda = 984/3.6 = 273.33$ *feet*

$\lambda/2a = 273.33/0.25 = 1093.33$

From the graph, $P = 0.956$. **Therefore, correct monopole length is** $(246 \times 0.956)/3.6 = 65.3$ *feet.*

to the total frequency span between the input impedance values as the 3-dB or half-power bandwidth of the antenna. This impedance behavior looks like this:

$$f_{low} = 36 - j36 = 50.91 \angle -45^\circ$$
$$f_o = 36 + j0 = 36.00 \angle 0^\circ$$
$$f_{high} = 36 + j36 = 50.91 \angle +45^\circ$$

Following good practice, let's select a transmission line for our monopole antenna whose uniform characteristic impedance, Z_o, is equal to that of the radiation resistance, R_r. This calls for a coaxial line of $Z_o = 36$ *ohms* (two 72-ohm coax lines connected in parallel). When we do this, at the resonant frequency at the band center, the vswr on the feedline will have a value of 1:1 because it is matched to its load impedance. When we tune the vfo to some frequency limit above or below resonance, $Z_{in} = 50.91 \angle \pm 45^\circ$ *ohms*.

At the two frequencies on either side of resonance where this impedance value is reached, the vswr in the feedline will have climbed to 2.6:1. Since the point where the absolute value of the reactive and resistive parts are equal, $|jX| = |R|$, marks the limits of the 3-dB bandwidth of the antenna, it is easy to wrongly assume that a vswr of 2.6:1 means that 3 dB or half the input power is reflected due to mismatch. However, a 2.6:1 vswr represents a mismatch condition where only 0.97 dB of the incident power is reflected. The reason for this much lower reflection is based on an important electrical law, Thevenin's theorem.*

The actual frequency width, $f_{high} - f_{low}$, at which the vswr rises to 2.6:1, however, depends upon the electrical nature of the antenna. To find out how to solve this problem, first write the quality factor Q for an antenna in terms of the 3 dB bandwidth as

$$Q = \frac{f_o}{f_{high} - f_{low}} \qquad \textbf{(1-7)}$$

All you need to solve the problem is a single design parameter of the antenna which is related to Q in frequency width. As you probably have guessed by now, this needed antenna design parameter is the mean characteristic impedance K_m of the monopole antenna. It is related to frequency Q as,

$$K_m = (46.5\ Q) + 3\ ohms \qquad \textbf{(1-8)}$$

Once you know these relationships, you can take the following design steps: First plug your particular amateur frequency limits, $f_{low} - f_{high}$, and f_o, into **eq. 1-7** to find Q; then plug the obtained Q value for the band into **eq. 1-8** and determine the needed K_m of the monopole. Then, all you have to do is solve for the antenna conductor length-to-radius ratio to give you this value of K_m.

To obtain this conductor size ratio, rewrite **eq. 2** of **table 1** in the following way,

$$\frac{K_m}{60} + 1 = log_e\ \frac{2(h_t')}{a'} \qquad \textbf{(1-9)}$$

Let's try this method out, first for the 10-meter band, and then for 80 meters. For 10 meters, the bandwidth limits are *29.7 MHz* to *28.0 MHz*. The band center of 28.85 MHz is our f_o. Solving for Q from **eq. 1-7**

$$Q = \frac{28.85}{29.7 - 28.0} = 16.97$$

$$K_m = (46.5 \ \ x\ 16.97) + 3 = 792\ ohms \qquad \text{(from \textbf{eq. 1-8})}$$

$$\frac{792}{60} + 1 = log_e\ \frac{2(h_t')}{a'} = 14.202 \qquad \text{(from \textbf{eq. 1-9})}$$

The needed conductor length-to-radius ratio is simply the natural antilogarithm obtained from **eq. 1-9**, or

$$\frac{2(h_t')}{a'} = 1.472\ x\ 10^6$$

$$a' = \frac{2(h_t')}{1.472\ x\ 10^6}$$

Here you need h_t' for the band f_o. Again start by *first assuming* that $h_t' = 0.250\lambda$ at 28.85 MHz (no correction as yet). Then $h_t' = 0.250\ x\ 34.11' = 8.53\ feet$; $2(h_t') = 17.05\ feet$.
Therefore,

$$a' = \frac{17.05'}{1.472\ x\ 10^6} = 1.16\ x\ 10^{-5}\ feet$$

$$= 1.39\ x\ 10^{-4}\ inches$$

conductor diameter $= 2.78\ x\ 10^{-4}$ *inches*

This is like shooting down a butterfly with a cannon! Here you went to some effort only to find that any gauge copper wire, physically strong enough to be suspended as an antenna, will give you the needed frequency vswr bandwidth on 10 meters. All this fancy frequency bandwidth design isn't for amateurs, right? To check on that, knowing how touchy Mother Nature can be, let's try the same exercise on the 80-meter band. It's width is 4.0 MHz to 3.5 MHz, with 3.75 MHz in the center. So

$$Q = \frac{3.75}{4.0 - 3.5} = 7.5$$

$$K_m = (46.5 \cdot 7.5) + 3 = 351.75\ ohms$$

$$\frac{351.75}{60} + 1 = log_e\ \frac{2(h_t')}{a'} = 6.86$$

Antilog (e) of 6.86 = 955.75

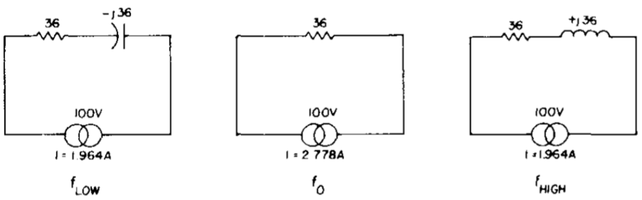
$$h_t'\ (0.250\lambda)\ at\ 3.75\ MHz = 65.6\ feet$$

$$a' = \frac{131.2'}{955.75} = 0.137\ feet = 1.65\ inches$$

conductor diameter is 3.29 inches

This is a horse of another color! You need an antenna

*Basically, Thevenin's theorem states that the current in any impedance connected to two terminals of a network is the same as if that impedance were connected to a generator whose voltage is equal to the open-circuited voltage at the terminals of the network. This can be illustrated by feeding an antenna with a constant-voltage source as shown below:



With a 100-volt constant-voltage source in series with the impedance, the current flow is 1.964 amp at f_{low} and f_{high}, and 2.77 amp at f_o. *The power dissipation in the real part of the impedance at the three* frequencies is as follows:

Power at f_{low} = *138.86 watts*
Power at f_o = *277.72 watts*
Power at f_{high} = *138.66 watts*

Note that the power difference is exactly 3 dB.

conductor whose minimum diameter is 3.3 inches to use in an antenna which will cover the entire 80-meter band without exceeding 2.6:1 vswr in the feedline. This means a length of 3.5 inch aluminum irrigation tubing; or triangular, but uniform diameter, triangular cross section tower at least 3.5 inch on each side for the 80 meter monopole.

For a doublet we can simulate the needed conductor diameter in the form of a lightweight cage design using a minimum of eight wires of say no. 18 AWG Copperweld. These wires can be arranged equally around the perimeter of circular plastic formers spaced along the length of each half of the doublet. At both ends of the dipole sections all wires should be brought to a point to form a taper where the insulators are attached. This is for reduction of shunt capacitance across the doublet input terminals. Excess shunt capacitance, either from the monopole conductor base to ground — or across the balanced doublet input terminals — will produce a non-symmetrical vswr curve at frequencies equally spaced above and below resonance. Reduction of base shunt capacitance is the reason you see those long tapers at the base of large cross section, vertical tower monopoles used by broadcast stations.

Finally, I promised to give a correction factor to take in the physical length of the monopole conductor, as a function of its diameter, so that it is ninety electrical degrees long at the resonant frequency. **Fig. 4** gives the correction curve and the detailed procedure needed to do this. When you actually design and put one of these *naturally resonant, broad-banded antennas on the air*, you will find that the vswr at the band edges will be somewhat less than 2.6:1. This comes about because of the addition of environmental ohmic losses induced in the antenna input impedance by the antenna site, as well as from the small decrease in antenna K_m when you *shorten the conductor to resonance by use of the data in* **fig. 4**.

In this article I have only scratched the surface in applying the analog key to the field of practical antenna design; to do a thorough job would require a book on the subject. Enough material has been given here, however, to enable you to quickly become proficient in the use of the analog key and to turn it loose on your own pet ideas for antenna design.

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